

# Spin Hall Conductance of the Two Dimensional Hole Gas in a Perpendicular Magnetic Field

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The charge and spin Hall conductance of the two-dimensional hole gas within the Luttinger model with and without inversion symmetry breaking terms in a perpendicular magnetic field are studied, and two key phenomena are predicted. The sign of the spin Hall conductance is modulated periodically by the external magnetic field, which means a possible application in the future. Furthermore, a resonant spin Hall conductance in the two-dimensional hole gas with a certain hole density at a typical magnetic field is indicated, which implies a likely way to firmly establish the intrinsic spin Hall effect. The charge Hall conductance is unaffected by the spin-orbit coupling.

PACS numbers: 73.43.-f, 72.25.Dc, 72.25.Hg, 85.75.-d

## INTRODUCTION

The intrinsic spin Hall effect (SHE) was predicted by Murakami *et al.*[1] in *p*-doped bulk semiconductors of a Luttinger Hamiltonian and by Sinova *et al.*[2] in two dimensional (2D) electron systems with Rashba spin-orbit coupling, and the intrinsic SHE has aroused intensive theoretical studies[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] for its potential applications in the emerging field of spintronics[17, 18, 19]. Moreover, experimental observation of the SHE has also been recently reported in an electron doped sample with the use of Kerr rotation microscopy[20, 21] and in a two-dimensional hole gas (2DHG) by angle-resolved polarization detection[22]. Kato *et al.*[20] suggested that the extrinsic[3] SHE in *n*-type semiconductor was dominant for that no marked crystal direction dependence was observed in the strained samples, while the SHE observed in the 2DHG with spin-orbit coupling was interpreted as intrinsic by Wunderlich *et al.*[4, 5, 6, 22]. Further experimental and theoretical work is needed to firmly establish the intrinsic VS extrinsic nature of the SHE. Besides, how to develop spintronics devices[17, 18, 19] is still a very challenging problem.

Although the SHE in the 2DHG has been intensively studied theoretically, most studies have been limited to zero external magnetic field[4, 5, 6, 7, 8, 9, 10, 11]. The spin Hall conductance in the 2DHG arises from the splitting between the light and heavy-hole band and the structural inversion asymmetry (SIA)[4, 8] in the 2DHG band. In this paper, we study transport properties of the 2DHG within the Luttinger[4, 23] model with and without inversion symmetry breaking terms in a perpendicular magnetic field, and two key phenomena are reported. First, the sign of the spin Hall conductance (*i.e.*, the direction of the spin Hall current) can be modulated periodically by the external magnetic field, which means a possible application in the future. Second, a resonant spin Hall conductance can be observed by adjusting the sample parameters and magnetic field, and the resonance may be used to firmly establish the intrinsic SHE in experiments[10]. Meanwhile, these two types of spin-orbit coupling have

no effect on the charge Hall conductance. The rest of the paper is organized as follows: The model Hamiltonian and the formula for charge and spin Hall conductance are introduced in section II. Our numerical result and discussion are shown in section III, and the paper is concluded with a summary in section IV.

## THEORETICAL FRAMEWORK

The effective Hamiltonian we consider for a 2DHG is a sum of both Luttinger and spin- $\vec{S} = 3/2$  SIA terms[4, 23]:

$$H_0 = \frac{1}{2m}(\gamma_1 + \frac{5}{2}\gamma_2)p^2 - \frac{\gamma_2}{m}(p_x^2 S_x^2 + p_y^2 S_y^2 + \langle p_z^2 \rangle S_z^2) - 2\frac{\gamma_2}{m}\{p_x, p_y\}\{S_x, S_y\} + \alpha(\vec{S} \times \vec{p}) \cdot \hat{z} \quad (1)$$

where  $\vec{S}$  is spin- $\frac{3}{2}$  operator and  $m$  is the bare electron mass, and we define  $\{A, B\} = \frac{1}{2}(AB + BA)$ ,  $p^2 = p_x^2 + p_y^2 + \langle p_z^2 \rangle$ . In addition,  $\gamma_1$  and  $\gamma_2$  are two dimensionless parameters modeling the effective mass and spin-orbit coupling around the  $\Gamma$  point. The confinement of the well in the  $z$  direction quantizes the momentum on this axis and it is approximated by the relation  $\langle p_z \rangle = 0$ ,  $\langle p_z^2 \rangle \approx (\pi\hbar/d)^2$  for a quantum well with thickness  $d$ . This is not the most general Hamiltonian when a magnetic field is present. The most general one involves both linear and cubic spin Zeeman terms, and we neglect these terms in Hamiltonian (1), which is a good approximation for GaAs[24, 25].

We impose a magnetic field  $\vec{B} = B\hat{z}$  by choosing the Landau gauge  $\vec{A} = -yB\hat{x}$ , and then  $p_x = \hbar k_x + \frac{eB}{c}y$ , where  $-e$  is the electric charge. The destruction operator[23]  $a = \frac{1}{\sqrt{2m\hbar\omega}}(p_x + ip_y)$ , and the creation operator  $a^\dagger = \frac{1}{\sqrt{2m\hbar\omega}}(p_x - ip_y)$  are introduced to describe Landau levels, where  $\omega = \frac{eB}{mc}$ . These operators have the commutation  $[a, a^\dagger] = 1$ . In terms of these operators and using explicit matrix notation with  $S = \frac{3}{2}$  eigenstates in the order  $S_z = +\frac{3}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ , Hamiltonian (1) within

a magnetic field can be rewritten as

$$H_0 = \hbar\omega \begin{pmatrix} H_{11} & i\sqrt{3}\eta a^\dagger & -\sqrt{3}\gamma_2 a^{\dagger 2} & 0 \\ -i\sqrt{3}\eta a & H_{22} & 2i\eta a^\dagger & -\sqrt{3}\gamma_2 a^{\dagger 2} \\ -\sqrt{3}\gamma_2 a^2 & -2i\eta a & H_{22} & i\sqrt{3}\eta a^\dagger \\ 0 & -\sqrt{3}\gamma_2 a^2 & -i\sqrt{3}\eta a & H_{11} \end{pmatrix},$$

with

$$\begin{aligned} H_{11} &= (\gamma_1 + \gamma_2)(a^\dagger a + \frac{1}{2}) + \frac{\beta}{2}(\gamma_1 - 2\gamma_2), \\ H_{22} &= (\gamma_1 - \gamma_2)(a^\dagger a + \frac{1}{2}) + \frac{\beta}{2}(\gamma_1 + 2\gamma_2). \end{aligned} \quad (2)$$

where the dimensionless parameters  $\eta = \alpha m \sqrt{\frac{c}{2\hbar e B}}$ , and  $\beta = \frac{\pi^2 \hbar}{d^2 m \omega}$ .

Depending on the confinement scale  $d$  the Luttinger term is dominant for  $d$  not too small, while the SIA term becomes dominant for infinitely thin wells, which corresponds to high junction fields. We first consider the case

when the confinement scale  $d$  is not too small, that is, neglect the SIA term:

$$\begin{aligned} H_0^L &= \frac{1}{2m}(\gamma_1 + \frac{5}{2}\gamma_2)p^2 - \frac{\gamma_2}{m}(p_x^2 S_x^2 + p_y^2 S_y^2 + \langle p_z^2 \rangle S_z^2) \\ &\quad - 2\frac{\gamma_2}{m}\{p_x, p_y\}\{S_x, S_y\}. \end{aligned} \quad (3)$$

The eigenstate of  $H_0^L$  can be taken as

$$|n, s, f\rangle = \begin{pmatrix} C_{nsf1}\phi_n \\ C_{nsf2}\phi_{n-1} \\ C_{nsf3}\phi_{n-2} \\ C_{nsf4}\phi_{n-3} \end{pmatrix}, \quad (4)$$

where  $\phi_n$  is the eigenstate of the  $n$ th Landau level in the absence of the Luttinger interaction, and the eigenvalues in units of  $\hbar\omega$  when  $n \geq 3$  can be obtained analytically as

$$E_{n+1sf} = (n + \frac{1}{2}\beta)\gamma_1 + \frac{f}{2}(\gamma_1 + 2\gamma_2) + s\sqrt{\gamma_1^2 + [2f(n - \beta) + 1]\gamma_1\gamma_2 + [(4n - 2\beta)(n + f) + f\beta + \beta^2 + \frac{1}{4}]\gamma_2^2}, \quad (5)$$

$$\theta_{n+1sf} = \arctan\left(\frac{-\sqrt{3}\gamma_2\sqrt{n(n+f)}}{[\gamma_1 + f(n - \beta)\gamma_2 + \frac{\gamma_2}{2}] + s\sqrt{\gamma_1^2 + [2f(n - \beta) + 1]\gamma_1\gamma_2 + [(4n - 2\beta)(n + f) + f\beta + \frac{1}{4}]\gamma_2^2}}\right). \quad (6)$$

The eigenvectors for  $s = \pm 1, f = 1$  can be expressed as

$$\begin{pmatrix} \cos \theta_{nsf} \\ 0 \\ \sin \theta_{nsf} \\ 0 \end{pmatrix}, \quad (7)$$

while those for  $s = \pm 1, f = -1$  can be expressed as

$$\begin{pmatrix} 0 \\ \cos \theta_{nsf} \\ 0 \\ \sin \theta_{nsf} \end{pmatrix}, \quad (8)$$

where  $\theta_{nsf}$  is defined in Eq.(6), and  $\theta_{n,1,1} = \theta_{n,-1,1} + \frac{\pi}{2}$ ,  $\theta_{n,1,-1} = \theta_{n,-1,-1} - \frac{\pi}{2}$ . The magnitude of  $\theta_{nsf}$  is important and we will discuss it corresponding to the parameters used in this paper below, and we can simply define that  $\theta_{n,-1,\pm 1} \in (0, \frac{\pi}{2})$  here. From large  $n$  limit, we can deduce that states  $|n, +1, \pm 1\rangle$  indicate light-hole bands (LH $^\pm$ ) and  $|n, -1, \pm 1\rangle$  indicates heavy-hole bands (HH $^\pm$ )[4, 25]. Besides, the eigenstates and eigenvalues for  $n < 3$  are easy to obtain and we do not show them explicitly here. The energy levels as functions of the filling factor  $\nu = \frac{N_h}{N_\phi} = \frac{n_h 2\pi \hbar c}{eB}$  are shown in Fig.1. We use different color lines to denote different  $n$  and

in order to give a more clear illumination below, we only plot the energy levels occupied for every  $\nu$ . Solid lines indicate Landau levels of mostly holes with spin- $\frac{3}{2}$  (states  $|n, -1, 1\rangle$ ) and dashed lines indicate Landau levels of mostly holes with spin- $-\frac{3}{2}$  (states  $|n, -1, -1\rangle$ ). From the bottom up,  $n$  increases one by one for solid lines (3~8) and dashed lines (0~5) respectively. Parameters used are taken from Ref.[22],  $n_h = 2.0 \times 10^{16}/m^2$ ,  $\gamma_1 = 6.92$ ,  $\gamma_2 = 2.1$ ,  $2\frac{\gamma_2}{m}(\frac{\pi \hbar}{d})^2 = 40\text{meV}$ , and  $d = 8.3\text{nm}$ . In addition,  $\alpha = 0$ . Inset: all the parameters used are the same except  $n_h = 2.73 \times 10^{16}/m^2$ . The dotted line indicates Landau levels of mostly holes with spin- $-\frac{1}{2}$ . The cross indicates the energy crossing which we will discuss below.

Let's begin to study the law in Fig.1 in detail (Don't include the Fig inset). When  $\nu_i = \text{odd}$  integers, the top energy levels occupied can be written as

$$E_{occupied}^{\nu_i=\text{odd}} = E_{n=\frac{\nu_i}{2}+\frac{5}{2}, -1, -1}, \quad (9)$$

while, when  $\nu_i = \text{even}$  integers, the top energy levels occupied are

$$E_{occupied}^{\nu_i=\text{even}} = E_{n=\frac{\nu_i}{2}-1, -1, 1}, \quad (10)$$

and the total energy levels occupied for every  $\nu$  include both states when  $\nu_i = \text{odd}$  and  $\nu_i = \text{even}$  where

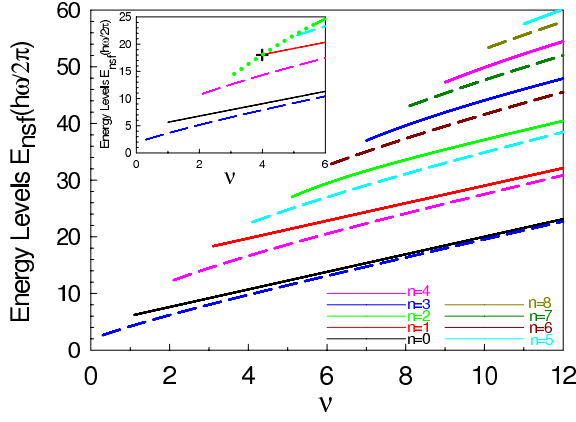


FIG. 1: (Color online) Landau levels (units:  $\hbar\omega$ ) as functions of filling factors  $\nu$ . Parameters used are taken from Ref.[4, 22] and  $\alpha = 0$ . Different colors denote different  $n$  and only energy levels occupied for corresponding  $\nu$  are shown. Solid lines indicate Landau levels of mostly holes with spin- $\frac{3}{2}$  and dashed lines indicate Landau levels of mostly holes with spin- $\frac{3}{2}$ . Inset:  $n_h = 2.73 \times 10^{16}/m^2$ . The dotted line indicates the Landau level of mostly holes with spin- $\frac{1}{2}$ . The cross indicates the energy crossing which gives rise to a resonance in spin Hall conductance.

$\nu_i = 1, 2, \dots, \nu$ . We should add that when  $n < 3$ , the definition of  $|n, s, f\rangle$  is not exact, so we simply take  $|2, -1, 1\rangle$  as the lowest energy level of  $n = 2$  and  $|1, 1, -1\rangle$  indicates the lowest energy level of  $n = 1$  in the whole paper. It is interesting to find that these energy levels arrange regularly, and the Landau levels of mostly holes with spin- $\frac{3}{2}$  and mostly holes with spin- $\frac{3}{2}$  appear by turns. This law is important and it will determine the behavior of the spin Hall conductance which we will discuss below.

Now we turn to study the system in the presence of an electric field applied in  $-\vec{y}$  direction. The full Hamiltonian  $H$  is

$$H = H_0^L + H', \quad (11)$$

with

$$H' = eEy = eE\sqrt{\frac{\hbar c}{2eB}}(a^\dagger + a) - \frac{E\hbar c}{B}k_x. \quad (12)$$

By treating  $H'$  as a perturbation term[13] up to the first order, we calculate the charge and spin Hall conductance along  $x$  direction, which is the most interesting case.

The charge current operator for a hole along  $x$  direction is given by  $j_c = ev_x$  with  $v_x = \frac{1}{i\hbar}[x, H]$ , and the spin  $S^z$  component current operator can be expressed as[12]

$$j_s = \frac{\hbar}{3}\{S^z, v_x\}. \quad (13)$$

From Eq.(13) and eigenstates (4) used, we can find that the contribution to the charge and spin Hall conductance arising from the zeroth order in  $H$  is zero for  $\langle n, s, f | j_{c,s} | n, s, f \rangle = 0$ .

We shall only focus on the first order in the perturbation in  $H'$ , which is

$$(j_{c,s})_{nsf} = \sum_{n's'f'} \frac{\langle n, s, f | j_{c,s} | n', s', f' \rangle}{\epsilon_{nsf} - \epsilon_{n's'f'}} \times \langle n', s', f' | H' | n, s, f \rangle + H.c., \quad (14)$$

where  $(j_{c,s})_{nsf}$  is the current carried by a hole in the state  $|n, s, f\rangle$  of  $H_0^L$ , and  $\epsilon_{nsf} = \hbar\omega E_{nsf}$ . In addition,  $n' = n \pm 1$ . The average current density of the system is given by

$$I_{c,s} = \frac{1}{L_y} \sum_{nsf} (j_{c,s})_{nsf} f(\epsilon_{nsf}), \quad (15)$$

where  $f(\epsilon_{nsf})$  is the Fermi distribution function, and the charge or spin hall conductance is

$$G_{c,s} = \frac{I_{c,s}}{EL_x}. \quad (16)$$

We can finally obtain the complete charge and spin Hall conductance, and the 2DHG system we consider is confined in the  $x - y$  plane of an area  $L_x \times L_y$ .

Our calculation of the charge and spin Hall conductance is complete for the linear response theory because that the zeroth order mean value of  $j_{c,s}$  is zero in each state  $|n, s, f\rangle$ , meanwhile we estimate the effect of relaxation to the Fermi distribution function  $f(\epsilon_{nsf})$  by means of Boltzman equation. The extra term of  $f(\epsilon_{nsf})$  from relaxation is proportional to the applied field, thus the effect on the conductance from relaxation is second order of the applied field  $E$ , so we can roughly neglect it at linear response region.

## RESULTS AND DISCUSSIONS

We calculate the charge Hall conductance at  $T = 0$  numerically firstly. Comparing with the usual quantum Hall effect case, *i.e.*,  $G_c = \nu \frac{e^2}{h}$ , our results show that the Luttinger type spin-orbit coupling has no effect on the charge Hall conductance, and this result is consistent with the quantization of the Hall conductance[26].

Now let's turn to discuss the magnetic field dependence of the spin Hall conductance. We calculate the spin Hall conductance within Hamiltonian (11), and the spin Hall conductance as a function of  $1/B$  is shown in Fig.2. In order to be convenient to illustrate it, Fig.2 is divided into two regions by a vertical green line at the value of magnetic field  $B = 20T$ . At the region that  $B < 20T$  (region 2), the sign of the spin Hall conductance changes periodically with the decreasing of magnetic field, and the periodicity is  $\frac{e}{n_h \hbar c}$ . The spin Hall conductance oscillates as a result of the alternative occupation of mostly holes with spin- $\frac{3}{2}$  and mostly holes with spin- $\frac{3}{2}$ . It reaches its maxima where  $\nu = \text{odd}$  and minima where  $\nu = \text{even}$ , while

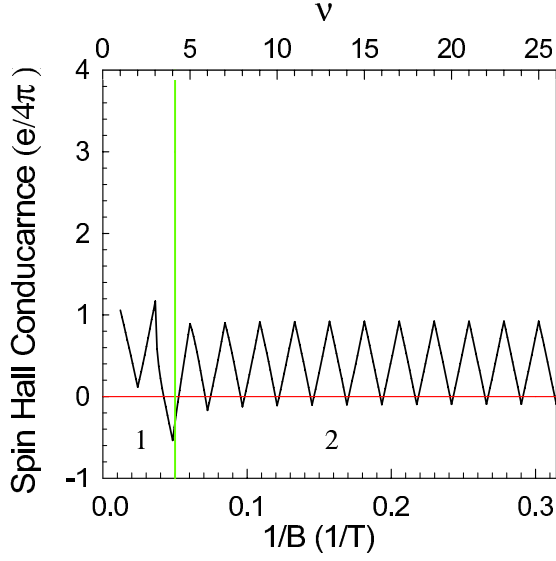


FIG. 2: (Color online) Spin hall conductance versus  $1/B$  at  $T = 0$ , and all the parameters used are the same as those in Fig.1.

the sign of the spin Hall conductance is opposite for odd and even  $\nu$ , except when  $\nu = 2$ .

Let's devote our attention to the region 1, especially where  $\nu$  changes from 3 to 5, and the corresponding  $B$  is  $16.5 \sim 27.6T$ , which can be achieved in laboratory now. When the absolute value of the spin Hall conductance is larger than 0.3 (*i.e.*, the spin Hall current is large enough to be detected) in the  $-x$  direction, the range of magnetic field is  $20.0 \sim 21.5T$ , while in the  $+x$  direction, the value of magnetic field is in the range of  $15.2 \sim 18.3T$ . The range of magnetic field for a notable spin Hall current is wide enough, which makes it easy to modulate the direction of the spin Hall current. If the direction of the spin Hall current can be detected, we can take the direction of the spin Hall current as a new sign, *i.e.*,  $+x$  direction means "0" and  $-x$  direction means "1", and these two states maybe the basic for a new logic electronic-device. In addition, we have made an scan on hole density, as well as magnetic field dependence of sign changes. We find that such rich sign changes are robust even in moderate magnetic fields, however, a more notable sign changes is sensitive to the hole density[25]. The sign issue should be regarded as a very important aspect of SHE and can be used in future experiments[25, 27].

Bernvig *et al.*[4] have calculated the spin Hall conductance within the same Hamiltonian (3) in the absence of a magnetic field, and their result shows that the spin hall conductance is  $\frac{1.9e}{8\pi}$ , which is in good agreement with the numerical estimate by Wunderlich *et al.*[22]. As shown in Fig.2, the jump in the spin Hall conductance tends to a stable value of  $\frac{2.0e}{8\pi}$ , which is very near to the universal value.

In the 2DEG with a Rashba spin-orbit coupling in a

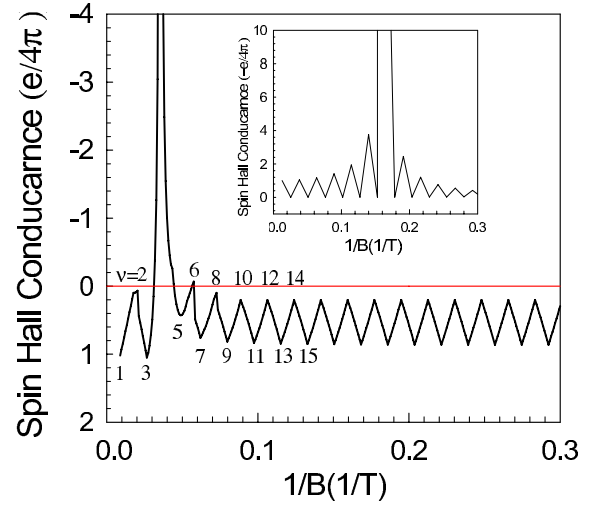


FIG. 3: (Color online) Resonant spin Hall conductance versus  $1/B$  at  $T = 0$ , and all the parameters used are the same as those in Fig.2 except  $n_h = 2.73 \times 10^{16}/m^2$ . Inset: Spin Hall conductance in 2DEG with a Rashba spin-orbit coupling versus  $1/B$  at  $T=0$ , and data are achieved within the method in Ref[13].

perpendicular magnetic field, the resonant spin Hall conductance stems from energy crossing of different Landau levels near the Fermi level[13]. Although this kind of energy crossing can appear when  $n_h = 2.0 \times 10^{16}/m^2$  of the sample we consider, but energy levels  $\epsilon_{n=1,1,-1}$  and  $\epsilon_{n'=n+1,-1,1}$  are higher than Fermi level as shown in Fig.1, so they do not contribute to the spin Hall conductance, and then there's no resonant spin Hall conductance when  $n_h = 2.0 \times 10^{16}/m^2$  as shown in Fig.2. If we modulate the hole density  $n_h$ , and take a value when the energy crossing occurs near the Fermi level as shown in Fig.1 inset ( $n_h = 2.73 \times 10^{16}/m^2$ ), there shall be an effective energy crossing between  $\epsilon_{n=1,1,-1}$  and  $\epsilon_{n'=n+1,-1,1}$  which occurs at  $B_r = 28.52T$  (the key point denoted by a cross in Fig.1 inset), we shall see a resonance in the spin Hall conductance as shown in Fig.3.

In order to make the energy crossing to be occurred near the Fermi level, it should request

$$3 < \frac{n_h 2\pi\hbar}{eB_r} < 4, \quad (17)$$

that is, when  $2.07 \times 10^{16}/m^2 < n_h < 2.75 \times 10^{16}/m^2$ , the "effective" energy crossing between mostly holes with spin- $-\frac{1}{2}$  and holes with spin- $-\frac{3}{2}$  shall give rise to a resonant spin Hall conductance in  $-x$  direction as shown in Fig.3. The resonant spin Hall conductance means spin accumulation[15, 16, 28] might be observed in experiments near the edges of a semiconductor channel, and it may be a way to distinguish the intrinsic from the extrinsic SHE[10].

Even all the results above are obtained numerically, we can discuss them in some analytical way. The

states near Fermi surface are mostly  $|n, -1, \pm 1\rangle$ . Let's argue the contribution for the spin Hall conductance when  $\nu_i = \text{odd}$  firstly, and the key states are  $|n, -1, -1\rangle$ . We can learn from the properties of eigenvectors shown in Eq.(7) and (8) that only states  $|n \pm 1, \pm 1, -1\rangle$  shall contribute to the spin Hall conductance for  $\langle n', \pm 1, +1 | H' | n, -1, -1 \rangle = 0$ , furthermore,  $|\epsilon_{n,-1,-1} - \epsilon_{n\pm 1, -1, -1}| \ll |\epsilon_{n,-1,-1} - \epsilon_{n\pm 1, 1, -1}|$ , so the spin Hall conductance arising from states  $|n, -1, -1\rangle$  shall be dominated by states  $|n \pm 1, -1, -1\rangle$

$$\begin{aligned} (j_S^{odd})_{n,-1,-1} &= \sum_{n's'f'} \frac{\langle n, -1, -1 | j_s | n', s', f' \rangle}{\epsilon_{n,-1,-1} - \epsilon_{n's'f'}} \\ &\times \langle n', s', f' | H' | n, -1, -1 \rangle + H.c. \\ &\approx \frac{E\hbar}{6B} \sum_{n', -1, -1} g_S^{odd}(n \rightarrow n'), \end{aligned} \quad (18)$$

and then all the part for  $\nu_i = \text{odd}$  can be obtained as

$$G_{\nu_i = \text{odd}} \approx \frac{e}{12\pi} g_S^{odd} \left( \frac{\nu_i}{2} + \frac{5}{2} + 1 \right) f(\epsilon_{\frac{\nu_i}{2} + \frac{7}{2}, -1, -1}), \quad (19)$$

where the Landau degeneracy factor  $m\omega/(2\pi\hbar)$  is also included and

$$\begin{aligned} g_S^{odd}(n \rightarrow n') &= \frac{6B}{E\hbar} \frac{\langle n, -1, -1 | j_s | n', -1, -1 \rangle}{\epsilon_{n,-1,-1} - \epsilon_{n', -1, -1}} \\ &\times \langle n', -1, -1 | H' | n, -1, -1 \rangle + H.c. \\ g_S^{even}(n \rightarrow n') &= \frac{6B}{E\hbar} \frac{\langle n, -1, 1 | j_s | n', -1, 1 \rangle}{\epsilon_{n,-1,1} - \epsilon_{n', -1, 1}} \\ &\times \langle n', -1, 1 | H' | n, -1, 1 \rangle + H.c. \end{aligned} \quad (20)$$

In deriving Eq.(18), we have used the fact that the transitions  $|i, -1, -1\rangle \rightarrow |i-1, -1, -1\rangle$  is canceled by  $|i-1, -1, -1\rangle \rightarrow |i, -1, -1\rangle$  where  $i \leq n$ , so only the  $g_S^{odd}(n \rightarrow n+1)$  is reserved and we simply write it as  $g_S^{odd}(n+1)$ . From Eq.(9), we can obtain that  $n+1 = \frac{\nu}{2} + \frac{5}{2} + 1$ . In the similar way, all the parts for  $\nu_i = \text{even}$  can be written as

$$G_{\nu_i = \text{even}} \approx \left[ \frac{e}{12\pi} g_S^{even} \left( \frac{\nu_i}{2} \right) f(\epsilon_{\frac{\nu_i}{2}, -1, +1}) \right]_{\nu_i \geq 4}, \quad (21)$$

and  $G_{\nu_i=2} = -\frac{e}{4\pi}$  can be calculated immediately. At zero temperature, when occupied, any  $f(\epsilon_{occupied}) = 1$ . The whole spin Hall conductance can be expressed as

$$\begin{aligned} G_{\nu = \text{odd}} &\approx \frac{e}{12\pi} \left[ g_S^{even} \left( \frac{\nu-1}{2} \right) + g_S^{odd} \left( \frac{\nu}{2} + \frac{7}{2} \right) \right], \\ G_{\nu = \text{even}} &\approx \frac{e}{12\pi} \left[ g_S^{even} \left( \frac{\nu}{2} \right) + g_S^{odd} \left( \frac{\nu}{2} + 3 \right) \right]. \end{aligned} \quad (22)$$

The magnitude of  $\theta_{n,-1,\pm 1}$  is important. Corresponding to the parameters used, and for the energy levels occupied of every  $\nu$ , we can define that  $\theta_{n,-1,1} \in (0, \frac{\pi}{4})$  and  $\theta_{n,-1,-1} \in (\frac{\pi}{4}, \frac{\pi}{2})$ . From the numerical calculation, we can obtain the following linear dependence of filling factor that when  $\nu = \text{odd}$ ,

$$\begin{aligned} g_S^{odd}(n) &\approx 1.12n - 2.07, g_S^{even}(m) \approx -1.12m - 0.97, \\ n &= \nu/2 + 5/2, m = (\nu-1)/2 - 1 \end{aligned} \quad (23)$$

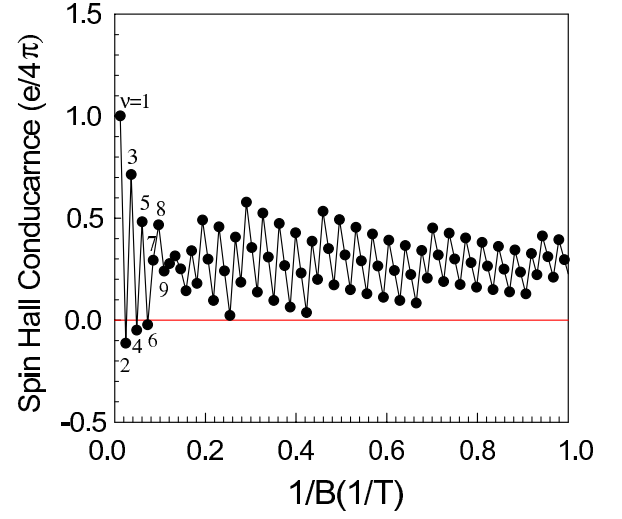


FIG. 4: (Color online) Spin Hall conductance versus  $1/B$  at  $T = 0$ , Parameters used are the same as those in Fig.1 except  $\alpha = 1.0 \times 10^5 \text{ m/s}$ .

and when  $\nu = \text{even}$ ,

$$\begin{aligned} g_S^{odd}(n) &\approx 1.12n - 2.04, g_S^{even}(m) \approx -1.12m - 0.88, \\ n &= (\nu-1)/2 + 5/2, m = \nu/2 - 1. \end{aligned} \quad (24)$$

From Eqs.(7) and (8), we can also learn that  $g_S^{odd}$  arises from the transition between mostly holes with spin  $-\frac{3}{2}$  and  $g_S^{even}$  arises from the transitions between mostly holes with spin  $-\frac{3}{2}$ . The oscillations on the spin Hall conductance due to the alternative occupation of mostly holes with spin  $-\frac{3}{2}$  and mostly holes with spin  $-\frac{3}{2}$  can be explained approximately, and our analytical process also indicates that the contribution from the transitions between mostly holes with spin  $-\frac{3}{2}$  (or spin  $-\frac{3}{2}$ ) should be the dominant part of the spin Hall conductance.

The contribution to the resonant spin Hall conductance can be obtained as

$$\begin{aligned} g_r &= [3\sqrt{2}(\gamma_1 + \gamma_2) \cos \theta_{2,-1,1} - 2\sqrt{3}\gamma_2 \sin \theta_{2,-1,1}] \\ &\times \frac{\sqrt{2} \cos \theta_{2,-1,1}}{E_{2,-1,1} - E_{1,1,-1}}, \end{aligned} \quad (25)$$

where  $\theta_{2,-1,1} = \arctan \frac{-\sqrt{6}\gamma_2}{[\gamma_1 + (\frac{3}{2} - \beta)\gamma_2] - \sqrt{[\gamma_1 + (\frac{3}{2} - \beta)\gamma_2]^2 + 6\gamma_2^2}}$ , and  $E_{2,-1,1} - E_{1,1,-1} < 0$  which leads to a resonant spin Hall conductance in  $-x$  direction. A more detailed discussion will be given in our further work[25].

The relatively large 5 meV measured splitting[4, 22] of the HH band implies that the effect of SIA terms is important. We have computed the spin conductance within the Hamiltonian (2) in the presence of an electric field in  $-\vec{y}$  direction, and the result is shown in Fig.4.

When  $\nu > 7$  (i.e.,  $B < 11.8T$ ), the spin Hall conductance oscillates as a result of the alternative occupation of mostly holes with spin  $-\frac{3}{2}$ , spin  $-\frac{3}{2}$  and spin  $-\frac{3}{2}$  between

some turning points,  $\nu = 7, 10, 37, 56, 76, 96$ , while the spin Hall conductance oscillates due to the alternative occupation of mostly holes with spin- $\frac{3}{2}$  and spin- $-\frac{3}{2}$  when  $B > 11.8T$ . Furthermore, the turning points appear periodically when  $B < 8.3T$ , and the period approaches a stable value  $19\frac{e}{n_h hc}$  when  $B < 2.2T$ . The difference between Fig.2 and Fig.4 can be found from the effect of SIA term. The SIA term arising from the structure inversion symmetry rearrange the energy levels when  $\nu > 7$ [25], and this effect is similar to the case in the absence of magnetic field[4, 10].

The charge Hall conductance has also been studied within the Hamiltonian (2) in the presence of an electric field in  $-\hat{y}$  direction, and our result shows that the SIA term has no effect on the charge Hall conductance. Our further calculation shows that we can also predict a resonant spin Hall conductance within the Luttinger model including the SIA term at a typical magnetic field[25] by adjusting the hole density.

After the completion of this paper, we become aware of the independent work[29] of M. Zarea and S. E. Ulloa, which studies the system of 2D heavy holes described by a  $k$ -cubic model in a magnetic field. In the thin quantum well limit, LH bands become energetically irrelevant and the HH bands can be effectively described by the 2D HH system[5, 8, 11, 30] where only the lowest HH subband is occupied. This condition can be satisfied when the quasi 2D system is sufficiently narrow and the density and the temperature are not too high[5, 11, 30]. Although both the paper by M. Zarea *et al.* and this paper by us compute the spin hall conductance in 2DHG in a magnetic field, M. Zarea *et al.*[29] focuses on the low field regime[30]. In strong magnetic field regime and at a high hole density, the resonance reported in this paper can not be achieved within the two dimensional HH system[25], which is due to the interplay between mostly holes with spin- $-\frac{1}{2}$  and holes with spin- $\frac{3}{2}$ . Moreover, the resonance remains within Luttinger model even including Zeeman splitting, and the resonance effect stemming from energy crossing of different Landau levels near the Fermi level due to the competition of Zeeman energy splitting and  $k$ -cubic spin-orbit coupling is out of reach for either real materials or in experiments[25]. However, when only the lowest HH subband is occupied, the 2D HH system can reproduce qualitatively the result within Luttinger model with Rashba spin-orbit coupling in the presence of a magnetic field[25].

## SUMMARY

In summary, we have studied the charge and spin Hall conductance in p-type GaAs quantum well structure described by a Luttinger Hamiltonian with the SIA term in a perpendicular magnetic field. The sign of the spin Hall conductance (*i.e.*, the direction of the spin Hall current)

purely caused by the Luttinger type spin-orbit coupling can be modulated periodically by the external magnetic field, which means a possible application in the future. The effective energy crossing between mostly holes with spin- $-\frac{1}{2}$  and holes with spin- $\frac{3}{2}$  at a typical magnetic field gives rise to a resonant spin Hall conductance with a certain hole density, and the jump in the spin Hall conductance is very near to the universal value of  $\frac{1.9e}{8\pi}$ . Our calculation shows that the hole density, the well thickness and the value of the magnetic field for the resonance are accessible in experiments. The resonance may be used to distinguish the intrinsic SHE from the extrinsic one. Although the SIA term rearranges the energy level of the Luttinger hamiltonian, the spin Hall conductance is similar to the case caused by pure Luttinger type spin-orbit coupling. The dominate contribution to the spin Hall conductance is the transitions arising from  $|n, -1, -1\rangle$  to  $|n \pm 1, -1, -1\rangle$  (between mostly holes with spin- $-\frac{3}{2}$ ) and  $|n, 1, -1\rangle$  to  $|n \pm 1, 1, -1\rangle$  (between mostly holes with spin- $\frac{3}{2}$ ). Our results also show that both Luttinger and spin- $\vec{S} = 3/2$  SIA spin-orbit coupling have no effect on the charge Hall conductance.

We thank Professors Shou-Cheng Zhang, Ruibao Tao, Shun-Qing Shen, Wuming Liu, Yu Shi and Dr. B. Andrei Bernevig for many helpful discussions.

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